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STUDY OF THE HYDRAULIC JUMP CHARACTERISTICS IN THE SMOOTH TRAPEZOIDAL CHANNEL

Field of engineering: Hydraulic Construction Engineering

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SUMMARY OF ENGINEERING DOCTORAL THESIS

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The thesis will be defended in front of the Institute-level Doctoral Thesis Judging Committee meeting at the Vietnam Academy For Water Resourses, at ....time ....day ...... month ....year 2022

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INTRODUCTION

1. Motivation of the doctoral thesis study

Hydraulic jump is a hydraulic phenomenon in an open-channel. In fact, the jump has many applications, typically the energy dissipation, the air entrainment etc.

Most studies on the jump carried out in the rectangular channel, but in the trapezoidal channel are relatively few, the equation system is still limited, many equations are still not suitable in the calculation.

In fact, the energy dissipation often uses the design method by a trapezoidal cross section, but the calculation method is still limited and has not yet guaranteed accuracy.

Therefore, "*Study of the hydraulic jump characteristics in the smooth trapezoidal channel*" has many scientific significances and is the basis for calculating and designing the constructions that it is applied the jump.

2. Goals and tasks of the study

The study goals: Establishing theoretical, semi-empirical and empirical equations for the hydraulic jump characteristics

The study tasks: Analyzing the geometrical features of the jump according to theory and using experimental data to test and develop new equations.

3. Objects and scope of the study

- *Objects of the study*: Studying the hydraulic jumpphenomenon.
- *Scope of the study*: Studying the steady jump ($Fr_{D1} = 4.0 \div 9.0$) in the horizontal trapezoidal channel (with a side slope m = 1:1).

4. Approaches and Methodologies of the study

The dissertation approach considers the theoretical analysis and the experimental research to establish new equations.

The research methodologies: Legacy data; Theory analysis and synthesis; Experimental methods; Statistical study; Professional solution; Dimensional Analysis; Numerical simulation.

5. Scientific and practical significance of the thesis

Scientific significance: Theoretical and experimental studies to set up the semi-empirical equations to calculate the sequent depths and the jump length in the trapezoidal channel.

Practical significance: The research result is the equations, it is used to calculate the geometrical characteristics of the jump.

6. New contributions of the thesis

+ Solving the Navier-Stokes differential equations, thereby determining the general equation (3.36) about the sequent depth ratio.

+ Analyzing the energy balance equation to establish the equation as a basis for the study of the roller length (3.40).

+ Study for the horizontal trapezoidal channel with a side slope m = 1. Determining the hydraulic characteristics of the jump as follows:

- The momentum coefficient ratio (k = 0.92). The Figure 3.5, Table 3.9 and the empirical equation (3.39) for finding the sequent depth.
- Establishing the semi-empirical equation (3.50) and experimental equation (3.53) for calculating the roller length.

7. Contents and structure of the thesis

The thesis has 03 chapters, in addition to the Introduction and Conclusion, illustrated with 46 tables, 82 figures and graphs, 06 related published publications (one paper is indexed in Scopus), 86 References and Appendices.

CHAPTER 1. OVERVIEW OF THE HYDRAULIC JUMP

1.1 Hydraulic jump in the trapezoidal channel

Hydraulic jump is a phenomenon in which the flow from a state with a depth smaller than the critical depth changes to a state with a depth greater than the critical depth.

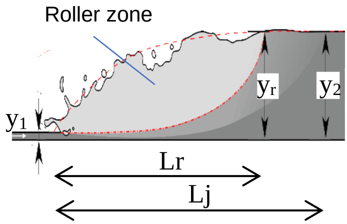


Fig. 1.3 Hydraulic jump

Given the complexity of the process of the roller zones, so the jump in the trapezoidal channel is more difficult to analyze than in rectangular channel.

1.5 Study of the hydraulic jump in the world

1.5.1 The phenomenon of the jump

The hydraulic jump was first proposed by Leonardo da Vinci (1452-1519). In 1818-1819, Giorgio Bidone described the jump.

1.5.2. Critical depth

The critical depth is an important parameter that determines the likelihood of the jump and the critical depth equation of the flow in the trapezoidal channel is the approximate equation.

There are different methods to determine the equation for calculating the critical depth of the trapezoidal channel, like Zhengzhong W. (1998), Tiejie C. et al. (2018), Farzin S. (2020) etc.

1.5.3 Hydraulic jump in the rectangular channel

a. Conjugate depths of the jump

In 1828, Bélanger proposed the equation for calculating the conjugate depths on a rectangular channel, this equation is still used in hydraulic documents of these days.

Based on experimental and theoretical research, other authors have also proposed equations to calculate the sequent depth, such as Sarma et al. (1975), Ead et al.(2002) and so on.

b. Length of the jump

The jump length is one of the characteristic parameters of the jump. The starting position or the toe of the jump was a consensus among the studies, but the end of the jump is not clear yet. In practice, the equations for calculating the length are mostly empirical ones.

Research on this issue can be mentioned: Riegel B. (1917), Woycicki (1931), Harry E.S et al. (2015), Martín M.M. et al.(2019) etc. There is a clear distinction about 2 types of the jump length: Hydraulic jump length (Lj) and Roller length (Lr). However, the relationship between Lj and Lr has not been studied clearly.

1.5.4 Hydraulic jump in the trapezoidal channel

a. Conjugate depths of the hydraulic jump

The study of the conjugate depths of the jump in the trapezoidal channel has been carried out by scientists through experimental, semi-empirical or theoretical methods, such as Wanoschek R. et al. (1989); Sadiq S.M. (2012); Bahador F.N et al. (2019) etc.

b. Hydraulic jump length

The studies of the jump length are mainly empirical equations, as the study of của Silvester, R. (1964), Ohtsu I. (1976), Afzal N. (2002), Kateb, S. (2014), Siad, R. (2018), Nobarian, B. F. et al. (2019) etc.

1.6 Study of the hydraulic jump in Vietnam

The researches on jumping water in Vietnam include Hoang Tu An (2005), Nguyen Van Dang (1989), Nguyen Thanh Don (2013), Le Thi Viet Ha (2018) and so on, but most of the studies are the hydraulic jump plane or the spatial jump. There have been no studies on the jump in the horizontal trapezoidal channel (semi-space).

1.7 Analyzing factors affecting geometrical features of the jump

1.7.1 Factors affecting the sequent depth

The analysis shows that the factors affecting the sequent depth of the jump depends on: Inflow Froude number (Fr_1 or Fr_{D1}); The velocity distribution; The roughness bed (e or n); The channel slope (i); The critical depth (y_c) etc.

1.7.1 Factors affecting the jump length

Analyzing of studies on the jump length in the channel, shows that the jump length depends on: The upstream depth of the jump (y_1); Downstream depth (y_2) of the jump; Height jump ($y_2 - y_1$); Conjugate depth ratio (y_2/y_1); Critical depth (y_c); Froude number (Fr_1 or Fr_{D1}) etc.

1.8 Conclusion for Chapter 1

The study has generalized the problems affecting the geometrical characteristics of the jump in the channels (rectangular and trapezoidal cross-sections), this is the research direction for a more complete analysis of the in the horizontal trapezoidal channel and evaluate the suitability of the new equations.

CHAPTER II - SCIENTIFIC BASIS AND RESEARCH METHODS DETERMINE THE CHARACTERISTICS OF THE HYDRAULIC JUMP IN THE TRAPEZOIDAL CHANNEL

2.1 Basic equation to determine the depth of the Roller

2.1.1 Basic differential equations of fluid motion

The Navier-Stokes equation is written as follows:

$$\overbrace{\rho \underbrace{\frac{\partial v}{\partial t}}_{\text{Gia tốc tức thời}} + \underbrace{v.\nabla v}_{\substack{\text{Gia tốc}\\ \text{đối lưu}}}}^{\text{Quán tính}} = \underbrace{-\nabla p}_{\substack{\text{Gradient}\\ \text{áp suất}}} + \underbrace{\mu.\nabla^2 v}_{\text{độ nhớt}} + \underbrace{f}_{\text{Lực}} \qquad (2.1)$$

Equation (2.1) is solved by specific boundary conditions.

2.1.2 Research hypotheses

+ The flow satisfies the continuum hypothesis and is steady; The law of pressure distribution obeys the hydrostatic rule; The force of gravity is oriented along the x-axis (Fx = gi);

+ Newtonian and incompressible fluids; Wet section is a symmetrical prism, The friction force is negligible.

2.1.3 Integrating differential equations of Navier-Stokes

Integrate the differential equation (2.1) on the cross-section A by integrating each term of the equation, get:

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\int_A u^2.dA-u_n\frac{\partial Q}{\partial x}-u_n^2\frac{\partial A}{\partial x}=\int_A F_x.dA-\frac{1}{\rho}\frac{\partial P}{\partial x}$$
$$-\frac{1}{\rho}\left(\frac{\partial T_{xx}}{\partial x}+\frac{\partial T_{xy}}{\partial x}+\frac{\partial T_{xz}}{\partial x}\right)-\frac{1}{\rho}\left(\int_A\frac{\partial\sigma_{xx}}{\partial x}dA-\tau_o A\right) \tag{2.14}$$

Applying the above assumptions to the equation (2.2), we have:

$$A\left(\frac{\alpha_o^*.V^2}{g}+z_c\right)=C_o \tag{2.17}$$

With C_o is an integral constant; α_o^* is a dynamic correction coefficient of flow with considering diffusion characteristics, flow friction, etc; A_i is a wet section (m^2); z_{ci} is a centroid depth (m).

Considering at two sections (1-1) and (2-2) of upstream and downstream in the roller of the hydraulic jump, there are:

$$A_1\left(\frac{\alpha_{01}^*V_1^2}{g}+z_{c1}\right)=A_2\left(\frac{\alpha_{02}^*V_2^2}{g}+z_{c2}\right) \tag{2.18}$$

Equation (18) used to calculate the depths of the hydraulic jump.

2.2 Basic equation in determining the roller length

2.2.1 Basic research hypothesis

+ Incompressible liquid, continuous motion; Steady Gradually Varied Flow; Horizontal channel (slope channel i = 0);

+ The hydraulic characteristics are analyzed according to an average from the depth y_1 to y_r (water surface, energy line...).

2.2.2 Equation for determining the roller length

Considering the energy balance equation of the flow is from section (1-1) to section (2-2), shown in Figure 2.1 and Figure 2.2.

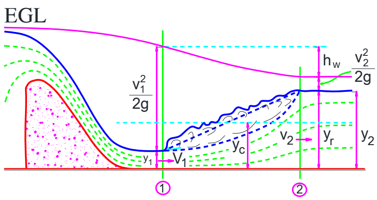


Figure 2.3 Diagram of the jump after the spillway

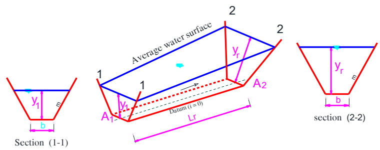


Figure 2.4 Diagram of analyzing the energy equation

The equation for determining the length of the jump is as follows:

$$L = \frac{\Delta E - \left(1 - A_1 / A_2\right)^2 \left(V_{tb}^2 / 2g\right)}{\left(Q / K_{tb}\right)^2} \qquad (2.23)$$

where:

h_{tb} is the average depth between the 2 sections upstream and downstream of the roller zone in the jump.

V_{tb}, K_{tb} is the velocity and discharge modulus in h_{tb}.

A_{tb}, C_{tb} và R_{tb} is the cross-sectional area, Chezy coefficient and hydraulic radius in h_{tb}.

ΔE is the energy dissipation of the jump (m), expressed as follows:

$$\Delta E = \left(y_1 + \alpha_1 V_1^2 / 2g\right) - \left(y_r + \alpha_2 V_2^2 / 2g\right) \qquad (2.24)$$

Equation (2.23) is used to determine the length of the gradually varied flow, which is applied as a basis to study the jump length.

2.3 Application of experimental zoning theory

2.3.1 Sequent depths of the jump

Using Pi theory for equation (2.18), general equation is as follows:

$$y_r / y_1 = \Psi\left(m.b / y_1, Fr_1, \sum \xi\right) = \Psi\left(M_1, Fr_1, \sum \xi\right) \qquad (2.42)$$

If m = 0, ignore $\left(\sum\zeta\right)$, so equation (2.8) became: $y_r/y_1 = \Psi(Fr_1)$, that the result is the Belanger equation (1828). Equation (2.42) has the influential factors the same to section 1.7

2.3.2 Hydraulic jump roller length

Using Pi theory for equation (2.23), the jump length is as shown:

$$\frac{Lr}{y_1} = \Phi\left(\frac{y_r}{y_1}, \frac{my_1}{b}, Fr_1, \frac{e}{y_1}\right) \qquad (2.56)$$

For a rectangular channel with a smooth bottom (e/y_1 = 0 and m = 0) the equation (2.56) remains: $Lr/y_1 = \Psi(y_r/y_1,\ Fr_1)$, that is similar to the studies Bélanger (1828). Equation (2.56) has fully shown the effects on the jump length (as analyzed in section 1.7).

2.4 Experimental setup

Experimental model was set up in the Vietnam Academy for Water Resources (KLORCE).

2.4.1 Structural model

The model consists of an ogee spillway, a straight-line trapezoidal channel is made of glass with the steel skeleton frame. It includes a hydrostatic tank, a an ogee spillway, a trapezoidal channel (a bed width 55cm and 33.5cm, 04 m length with a side slope m = 1:1), measuring water level and control downstream water level.

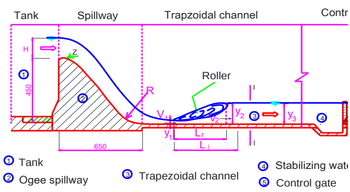


Figure 2.11 Plan of the experimental equipment

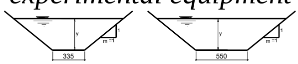


Figure *2.13 Cross-sections*

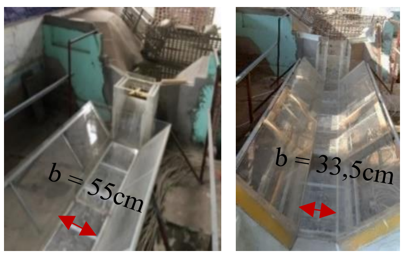


Figure *2.15* Experimental model

2.4.2 Method of **experiment and data processing**

2.4.2.1 Experimental Case Studies

From the analysis of the relationship between the hydrodynamic characteristics of the jump by Pi theory, it is shown that the quantities to be collected in each experimental case, as follows:

Table 2.4 Measurement parameters on physical experimental model

No.	Parameters	Symbol	Unit	Notes
1	Discharge	Q	m^3/s	Meter flume
2	Initial depth	y_1	m	Measuring by a levelling staff and a surveying equipment.
3	Sequent depth	y_r	m	
4	Roller length	L_r	m	Measuring by a ruler

2.4.2.2 Experimental data

The experimental values are shown in Table 2.5, as follows:

Table 2.5 Experimental data range

Values	Water level gauge (cm)	Q (m^3/s)	b (cm)	Geometric features		
				y_1 (m)	y_r (m)	L_r (m)
Max	47.55	0.201	55	0.092	0.488	2.1
Min	30.2	0.04	33.5	0.04	0.182	0.8

From data in Table 2.5 shows that the flow rate Q = 40 l/s ÷ 201 l/s with Fr_{D1} = 4.0 ÷ 8.4, the value of the roller length within the study channel. (channel length L = 4m), the matched dataset has 34 cases.

In this study, the experimental series is a combination of parameters: Q, y_1, y_2 and Lr. Experimental analysis according to the total factor, the minimum number of experiments to be performed is $2^m = 2^4 = 16$ (m is the number of influential factors). Thus, the experimental data meet the analysis process of changing the geometrical characteristics of the steady jump.

2.5 Experimental relationship between hydrodynamic characteristics in the hydraulic jump

Based on experimental data, the relationship between the hydrodynamic characteristics of the jump was analyzed to evaluate the correlation relationship between the influencing factors.

+ Relationship between the inflow Froude number (Fr_{D1}) and the sequent depth: the relationship between (y_r/y_1) with Fr_{D1}, it shows a high correlation relationship (R^2 = 0.95), which demonstrates the close dependence of the sequent depth on the Froude number.

+ Effecting on the roller length: Factors such as the ratio of the sequent depth, Froude number, energy... have a relationship to the roller length, it shows a high correlation relationship (R^2 = 0.95). The analysis also shows that the ratio Lr/y_1 used to study the roller length (Lr) is appropriate.

2.6 Conclusions of Chapter II

In this chapter, the thesis presents the scientific basis in studying on the geometrical features of the jump from basic equations.

Building experimental models, measuring data and evaluating data show the assurance of research conditions on the trend of changing geometrical characteristics of the jump in the trapezoidal channel.

CHAPTER III - ESTABLISHING EQUATIONS FOR DETEMAINING GEOMETRICAL FEATURES OF THE HYDRAULIC JUMP IN THE TRAPEZOIDAL CHANNEL

3.1 Establishing a critical depth equation

From theoretical analysis and data of the critical depth, the equation for determining the critical depth (y_c) of the flow in an isosceles trapezoidal channel ($m \neq 0$) has been developed:

$$y_c = \frac{13.b}{10.m}\left(\sqrt{1+\frac{20m}{13b}.y_{cCN}} - 1\right) \quad (3.8)$$

where: y_{cCN} is a critical depth of the rectangular channel

Evaluating equation (3.8) according to the statistical criteria:

Table 3.2 Calculation of the critical depth

Values	**Sample value**					**Equation (3.8)**		
	Q (m^3/s)	b (m)	m	y_c (m)	y_{cCN} (m)	M_c	$y_{c\ Pre}$ (m)	ε (%)
Max	100	10	2	2.727	3.442	0.481	2.727	0.37
Min	1	1	0.5	0.28	0.294	0.019	0.279	0

Table 3.3 Statistical indicators

Equation	**MAE**	**MSE**	**RMSE**	**R^2**	**MAPE (%)**
CT 3.8	0.002	0.000	0.003	0.999	0.188

Based on the results in Table 3.2, it shows that the equation (3.8) gives very good results, this is reflected in the error is less than 0.37%, the $R^2 \approx 1$ and other statistical indicators are approximately zero.

3.2 Establishing an equation of the sequent depth

3.2.1 General formula for determining the sequent depth

The mathematical transformation (2.18) for the case of the jump in the isosceles trapezoidal channel, it is obtained as follows:

$$M_1^2Y^4 + (M_1^2 + \frac{5}{2}M_1)Y^3 + (M_1^2 + \frac{5}{2}M_1 + \frac{3}{2})Y^2 + \left[-3F_{rD1}^2.\frac{(M_1+1)^2}{2M_1+1}.M_1 + M_1 + \frac{3}{2}\right]Y - 3kF_{rD1}^2.\frac{(M_1+1)^3}{2M_1+1} = 0 \quad (3.32)$$

where: M_1: Side wall constant in trapezoidal channel, $M_1 = m.y_1/b$

Y: Sequent depth ratio, $Y = y_r/y_1$

k: The momentum coefficient ratio of the jump, $k = \alpha_{02}^*/\alpha_1$

If $M_1 = 0$, $k \approx 1$, the equation (3.32) will become Bélanger equation.

Solving equation (3.32) determines the sequent depth ratio according to the appropriate equation as follows:

$$Y = \frac{-a}{4} + \frac{1}{2}\sqrt{f+v+z} + \frac{1}{2}\sqrt{f-v-z+\frac{q}{4\sqrt{f+v+z}}} \qquad (3.36)$$

where: $a = (1+\frac{5}{2M_1})$ $\quad c = -3Fr_{D1}^2.\frac{(M_1+1)^2}{(2M_1+1)M_1} + \frac{1}{M_1} + \frac{3}{2M_1^2}$

$$b = 1 + \frac{5}{2M_1} + \frac{3}{2M_1^2} \qquad d = -3kFr_{D1}^2.\frac{(M_1+1)^3}{(2M_1+1)M_1^2} \qquad f = \frac{a^2}{4} - \frac{2b}{3}$$

$$w = b^2 - 3ac + 12d \qquad q = -a^3 + 4ab - 8c \qquad s = 2b^3 - 9abc + 27c^2 + 27a^2d - 72bd$$

$$v = \frac{2^{1/3}.w}{3\left(s+\sqrt{-4.w^3+s^2}\right)^{1/3}} \qquad z = \left(\frac{s+\sqrt{-4.w^3+s^2}}{54}\right)^{1/3}$$

Equation (3.36) is used to calculate the sequent depth ratio of the jump in the trapezoidal channel.

Studying with M_1 = (0 ÷1), k = 1 and Fr_{D1} =(3,5 ÷ 10), the equation (3.36) is done in Fig. 3.4.

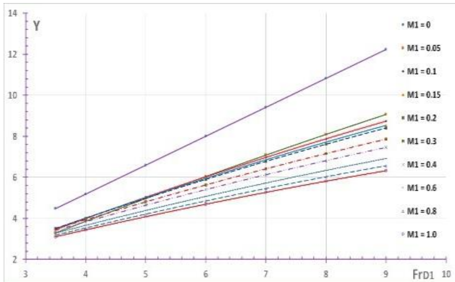


Figure 3.4 Relationship between Y on M_1 and Fr_{D1}

When $M_1 \geq 0.2$, the law by changing the ratio of sequent depth is quite uniform in the case of steady jump (the same to Hager W. (1992). When the M_1 is from 0.0 to 0.2, equation (3.36) is no longer relevant.

3.2.2 Determining ratio of the momentum coefficient

To determine the coefficient k, the study will use current experimental data and data of Wanoschek R. & Hager W. (1989) for research. Research data must satisfy the following conditions:

$$\{4,0 \leq Fr_{D1} \leq 9,0; \quad M_1 \geq 0,2; \quad y_1 \geq 3cm; \}$$

After filtering according to the proposed condition, the remaining 22 cases are shown in Table 3.6:

Table 3.6 Experimental data on the sequent depth

Values	Q (m^3/s)	y_1 (m)	y_r (m)	Fr_{D1}	M_1	$Y_{td} = y_r/y_1$
Max	0.158	0.092	0.448	8.681	0.406	7.922
Min	0.0242	0.0405	0.17	3.636	0.2	3.556

From equation (3.36), the ratio of the sequent depth is calculated with the following cases:

$$\begin{cases} Fr_{D1} = 4;\ 5;\ 6;\ 7;\ 8;\ 9;\ 10. \\ M_1 = 0,2;\ 0,3;\ 0,4;\ 0,5;\ 0,6;\ 0,7;\ 0,8;\ 0,9;\ 1,0. \\ k = 0,9;\ 0,91;\ 0,92;\ 0,94;\ 0,95;\ 0,97;\ 1;\ 1,045;\ 1,56. \end{cases} \tag{3.37}$$

Thus, for each combination (Fr_{D1}, M_1) in Table 3.3 and a value (k) in condition (3.37), a value (Y_{tt}) will be calculated according to Eq. (3.36). Then, evaluating between the measured values (Y_{td}) and the calculated values (Y_{tt}). The results are shown in the Table 3.7 and 3.8.

Table 3.7 Calculating the sequent depth according to the equation (3.36)

Values	Fr_{D1}	M_1	Y_{td}	k = 1		k = 0.92		k = 0.91	
				Y_{tt}	ε %	Y_{tt}	ε %	Y_{tt}	ε %
Max	8.681	0.406	7.922	7.980	5.3	7.856	4.0	7.840	3.8
Min	3.636	0.200	3.556	3.524	0.3	3.463	0.3	3.455	0.1

Table 3.8 the statistical indicators in studying a coefficient k

TT	k	MAE	MSE	RMSE	R^2	MAPE (%)
1	1	0.111	0.021	0.146	0.982	2.034
2	0.93	0.078	0.011	0.106	0.990	1.453
3	0.92	0.079	0.011	0.104	0.991	1.498
4	0.91	0.081	0.011	0.105	0.990	1.546
Max	1	0.111	0.021	0.146	0.991	2.034
Min	0.9	0.078	0.011	0.104	0.982	1.441

From the statistical indicators in Table 3.8, it shows that, k = 0.92, the theoretical equation (3.36) with 2 parameters (R^2 and RMSE) is better than the remaining cases.

3.2.3 Establishing a method to determine the sequent depth

Finding the sequent depth from the equation (3.36), if $M_1 < 0.2$, then the value calculated from the equation (3.36) is not stable. Dealing with this phenomenon, if the M_1 is in the range (0.0 ÷ 0.2), then Y will be interpolated according to the analytical solution data values in Table 3.4. The interpolation analysis with k = 0.92 are in Table 3.9:

Table 3.9 Values of the sequent depth ratio with k = 0.92

Fr_{D1}	M_1												
	0	**0.05**	**0.1**	**0.15**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
4	5.18	4.53	4.17	3.99	3.90	3.86	3.77	3.69	3.62	3.55	3.49	3.45	3.40
5	6.59	5.78	5.30	5.03	4.87	4.73	4.58	4.45	4.34	4.25	4.17	4.11	4.05
6	8.00	7.00	6.38	6.01	5.78	5.55	5.33	5.16	5.02	4.90	4.81	4.72	4.65
7	9.41	8.20	7.43	6.96	6.65	6.32	6.05	5.84	5.66	5.52	5.41	5.31	5.22
8	10.82	9.38	8.45	7.86	7.48	7.06	6.73	6.48	6.28	6.11	5.98	5.86	5.76
9	12.24	10.54	9.44	8.74	8.28	7.77	7.39	7.10	6.86	6.68	6.52	6.39	6.28

Table 3.9 is used to determine the sequent depth of the jump according to the values (Fr_{D1} and M_1), which are shown in Figure 3.5.

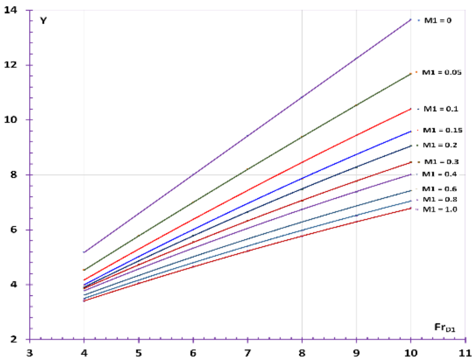


Figure 3.5 Determining Y according to (Fr_{D1} and M_1) when k = 0.92

Thus, the sequent depth ratio of the jump in the horizontal trapezoidal channel determined according to Table 3.9 or the Figure 3.5 will be more suitable, because it is true for all M_1 = 0.0 ÷ 0.2 and Fr_{D1} = 4.0 ÷ 9.0.

3.2.4 An empirical sequent depth equation

From formulas (2.42) and (3.36), empirical equation for determining the sequent depth of the hydraulic jump as follows:

$$\frac{y_r}{y_1} = 0,959 M_1^{-0,12} Fr_1^{0,936} \qquad (3.39)$$

Evaluating the equation (3.36) and the equation (3.39) for all data, some statistical indicators when comparing the measured data and calculated values. It is shown in Table 3.15:

Table 3.15 Statistical indicators for the data of the sequent depth

No	Equations	MAE	MSE	RMSE	R^2	MAPE (%)
1	Eq.(3.36)	0.006	0.000	0.008	0.991	1.725
2	Eq.(3.39)	0.009	0.000	0.012	0.978	2.776

From Table 3.15, it can be seen that the correlation relationship between the measured values and calculated values according to the theoretical method is very high, shown in R^2 = 0.99 and other statistical indicators are very small. However, both equations have their own meaning in determining the sequent depth, in practice it is recommended to use both equations to verify the calculated results.

3.3 Establishing an equation of the hydraulic jump roller length in the horizontal trapezoidal channel

3.3.1 Analysis and evaluation of the roller length equation

Dividing both sides of the equation (2.23) by y_1, the ratio of the length to the depth is determined as follows:

$$M_n = \frac{L}{y_1} = \frac{1}{y_1}\frac{\Delta E - (1 - A_1 / A_2)^2 (V_{tb}^2 / 2g)}{(Q / K_{tb})^2} \qquad (3.40)$$

The Mn value is a dimensionless coefficient and transforming the equation (3.40), get:

$$M_n = \frac{1}{(Q/K_{tb})^2}\left[\left(1-\frac{y_r}{y_1}\right)+\frac{1}{2}\left(1-\frac{A_1^2}{A_2^2}\right)\left(\frac{M_1+1}{2M_1+1}\right)Fr_{D1}^2-\frac{1}{y_1}\left(1-\frac{A_1}{A_2}\right)^2\frac{V_{tb}^2}{2g}\right] \quad (3.49)$$

Equation (3.40), (3.49) is a length equation with steady and gradually varied flow. Therefore, the Mn will be many times larger than the actual measured of the hydraulic jump roller length (Lr/y_1), so this relationship will be confirmed by experimental research.

3.3.2 Establishing the semi-empirical equation to estimate the roller Length of the hydraulic Jump

Based on the experimental data in Table 2.5 and the data of Wanoschek R. & Hager W. (1989), the research data are gathered into a set of analytical data and test data. Using Eq. (3.40), calculating the components according to the analytical data as follows:

Table 3.17 Analyzing the experimental data for the roller length

No	Lr/y_1 đo	K_{tb} (m^3/s)	ΔE (m)	Mn	ln(Mn)
Max	39.583	6.824	1.475	35267	10.471
Min	17.263	0.517	0.173	1729	7.455

The relationship between ln(Mn) and Lr/y_1 is in Fig.(3.9):

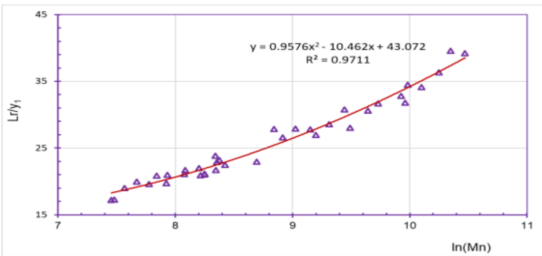


Figure 3.9 Relationship between ln(Mn) and the ratio (Lr/y_1)

The equation for the roller length is shown as follows:

$$\frac{Lr}{y_1} = 0{,}9576[\ln(Mn)]^2 - 10{,}462\ \ln(Mn) + 43{,}072 \quad (3.50)$$

trong đó: Mn được tính theo công thức (3.49)

where: Mn is calculated according to the equation (3.49)

Equation (3.50) is used to determine the roller length of the steady jump (FrD1 = 4.0 ÷ 9.0) in the isosceles trapezoidal channel, with a stilling basin and a side slope m = 1.

3.3.3 An empirical equation of the roller length

From formulas (2.46) and (3.50), an experimental research on reducing the semi-empirical equation for the roller length is shown:

$$\frac{Lr}{y_1} = 3{,}038\left[\left(\frac{y_r}{y_1}\right)^{0{,}94} + \left(1 - \frac{A_1^2}{A_2^2}\right) Fr_1^{0{,}683} + 0{,}578\right] \tag{3.53}$$

with condition: $e/y_1 = 1/120 \div 1/80$ (3.54)

Testing equations (3.50) and (3.53) according to the following statistical indicators in Table 3.21:

Table 3.22 Statistical indicators for the data of the roller length

No	Equations	MAE	MSE	RMSE	R^2	MAPE (%)
1	Eq. (3.50)	0.064	0.005	0.072	0.962	4.365
2	Eq. (3.53)	0.070	0.006	0.075	0.959	4.628

As shown in Table 3.22, showing that the equations all give very good statistical evaluation criteria, the coefficient $R^2 \geq 0.96$, the other indicators come close to zero.

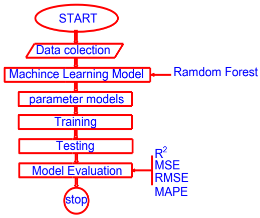
3.5 Prediction of the geometric features of the hydraulic jump using Random Forest regression

3.5.1 Structure of machine learning model

Using Matlab software to build a "Random Forest" model according to Friedman's Least-Squares Boosting (LS_Boost) algorithm (2001).

The parameter estimation process is based on two main steps:

- Step 1: Train the Random Forest model.
- Step 2: Estimate the parameters based on the trained model.



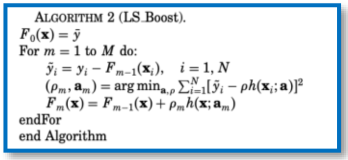


Figure 3.12. Flowchart of the Random Forest Model

Figure 3.13. Least-Squares Boost algorithm in Friedman (2001

The Random Forest model was implemented in Matlab 2019b.

3.5.2 Prediction of the sequent depth

From Eq.(2.42), establishing the data fields of the Machine Learning method are as follows:

Table 3.23 Data for analyzing the sequent depth

No.	Parameters	Max	Min	Notes
1	y_r/y_1	9.396	3.847	Target variable
2	$M_1 = my_1/b$	0.406	0.073	Analytical data variables
3	Fr_1	9.315	3.639	

Table 3.24 Prediction of the sequent depth by the test data

Values	Mearsured values			Predicted values			
				Decision Tree		Random Forest	
	Fr_1	M_1	y_r/y_1	y_r/y_1	ε (%)	y_r/y_1	ε (%)
Max	6.30	0.41	6.82	6.52	8.3	6.35	7.9
Min	3.99	0.11	4.26	4.4	1.5	4.19	0.4

The calculation error of the test data shows that, the maximum error is about 7.9%, this value is higher than the error calculated according to the theory method (5.4%) and the empirical equation (6.8%)).

Table 3.27 Statistical indicators for the test data of the sequent depth

Parameter	MAE	MSE	RMSE	R^2	MAPE (%)
Sequent depth	0.190	0.058	0.242	0.905	3.681

Thus, the calculation according to the "Random forest" model with the LS_Boost algorithm of Breiman, L (2001) gave a good efficiency in analyzing the sequent depth of the jump.

3.5.3 Prediction of the roller length

From Eq.(2.42), establishing the data fields of the Machine Learning method are as follows:

Table 3.25 Data for analyzing the Roller length

No.	Parameters	Max	Min	Notes
1	Lr/y_1	39.583	17.263	Target variable
2	M_1	0.406	0.073	Analytical data variables
3	Fr_1	9.32	3.64	
4	y_r/y_1	9.396	3.847	

Table 3.26 Prediction of the roller length by the test data

Values	Mearsured values				Predicted values	
	Lr/y_1	Fr_1	M_1	y_r/y_1	L_r/y_1	ε (%)
Max	**27.69**	**6.30**	**0.41**	**6.82**	**28.08**	**7.03**
Min	**17.74**	**3.99**	**0.11**	**4.26**	**16.94**	**0.11**

The Predicted values with measured data has the largest error of 7.0% (while the error of the semi-empirical and experimental equations for the jump length is 8.0% and 7.5% respectively).

Table 3.27 Statistical indicators for the test data of the roller length

Parameter	MAE	MSE	RMSE	R^2	MAPE (%)
Rolle length	0.802	0.884	0.940	0.900	3.527

Analyzing and evaluating of statistical indicators for the test data according to "Random Forest" model, showed that $R^2 \geq 0.9$ (strong correlation), other statistical indicators are also close to 0. This shows that the efficiency of the model is very good.

The study has obtained the initial basic data set for the study of the jump in the trapezoidal channel. Obtained results on the geometric features from the "Random Forest" have shown the small error and the agreement of the measured and calculated values.

3.6 Calculation process to determine the hydraulic jump characteristics in the design of energy dissipation construction

Combining analysis, research and calculation steps of the geometrical features of the jump in a horizontal trapezoidal channel (with m = 1).

The procedure for determining the geometrical dimensions in the design of a dissipation tank is shown in Figure 3.24.

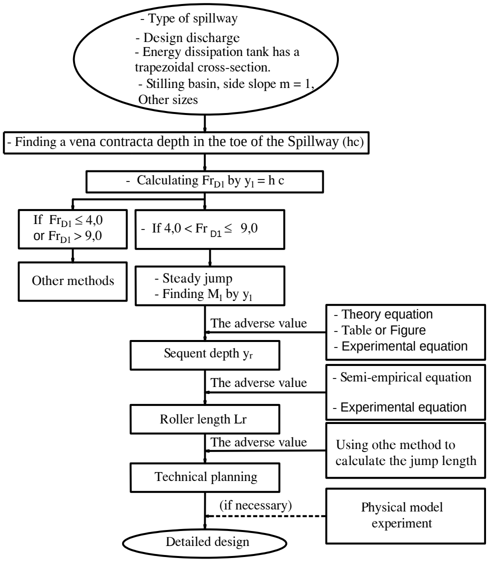


Figure 3.24 The process of designing a energy dissipation by the steady jump

3.7 Using proposed equations to calculate the geometric features of the jump after the spillway of Na San reservoir (Son La)

***3.7.1 Introduction to* Na San reservoir**

+ Name: Na San irrigation system.

+ Address: Mai Son district, Son La province.

+ Objective: Ensure water supply for agricultural production through investment in irrigation system infrastructure and combined with creating domestic water supply for Na San plateau.

Table 3.30 Spillway's Parameters of Na San reservoir

No.	Spillway and crest gate	Ogee spillway	
1	Spillway structure	Concrete gravity dam	
2	Elevation of dam crest	m	+762,0
3	Design flood discharge $Q_{tk\ 1\%}$	m^3/s	426,3

3.7.2 Determining design parameters of an energy dissipation construction

Based on the design document, the basic input parameters are as follows:

Table 3.31 Parameters for calculating the energy dissipation construction

No.	Parameters	Symbols	Unit	Value	Notes
1	Design discharge	Q	m^3/s	426.3	P = 1%
2	Vena contracta depth	h_c	m	0.84	
3	Energy dissipation		Trapezoidal channel		
4	Side slop of channel	m		1	
5	Bed width	b	m	20.5	
6	Material				Concrete
7	Manning coefficient	n		0.018	Concrete

3.7.3 Calculating the geometrical parameters of hydraulic jump for the energy dissipation construction of Na San reservoir

3.7.3.1 Determining sequent depth

The basic parameters at the toe of the spillway are determined according to the following table:

Table 3.32 Hydraulic parameters at a vena contracta depth

Q (m^3/s)	B (m)	m	y_1 (m)	Fr_1	Fr_{D1}
426.3	20.5	1	0.84	8.29	8.45

Table 3.32 shows that, the inflow Froude number $Fr_{D1} = 8.45$, so the hydraulic jump in the trapezoidal channel is the steady jump.

The depth at the end of the roller zone of the jumps according to the options as in Table 3.33:

Table 3.33 the sequent depths according to methods

Q	y_1	Table (3.8)		Eq. (3.39)		Machine Learning	
(m^3/s)	(m)	(y_r/y_1)	y_r (m)	(y_r/y_1)	y_r (m)	(y_r/y_1)	y_r (m)
426.3	0.84	10.18	8.55	10.18	8.55	9.4	7.9

Selected design the sequent depth: $y_r = 8.55$ (m)

3.7.3.2 Calculation of roller length

From the sequent depth of the jump is calculated above, applying equations to determine the roller length, the results of case studies and calculated values are shown in Table 3.34 as follows:

Table 3.34 Calculation of the roller length

Eq. (3.50)			Eq. (3.53)		Machine Learning	
Ln(Mn)	Lr/y_1	Lr (m)	Lr/y_1	Lr (m)	Lr/y_1	Lr (m)
10.35	39.21	32.9	39.78	33.42	39.52	33.2

Calculation results according to semi-empirical equation, empirical equation and Machine Learning models are the same. The calculation results according to the semi-empirical equation has the largest value, so the roller length is determined: Lr = 34(m)

3.7.3.3 Energy dissipation

The energy dissipated by the jump in the isosceles trapezoidal channel (side slope m = 1) is determined as follows in Table 3.36:

Table 3.36 analysis of the energy dissipation for the jump

Q (m^3/s)	y_1(m)	y_r(m)	E_1(m)	E_2(m)	ΔE (%)
426.3	0.84	8.55	29.666	8.707	70.6

Steady jump after the spillway has a great energy dissipation efficiency (up to 70.6%) and this dissipation has reached the energy dissipation range of a strong hydraulic jump.

3.8 Conclusion of Chapter III

The study has determined the theoretical equation to determine the sequent depth of the roller zone in the jump. With the trapezoidal channel (the flat bottom, a side slope of m = 1 and the k = 0.92) has been determined the empirical equation for the sequent depth, the semi-empirical and experimental equation for the roller length (Lr).

The initial application of machine learning model "Random Forest" gives good results in analyzing the geometrical features of the jump.

The process of determining the geometrical dimensions in the design of the dissipation tank has an isosceles trapezoidal cross-section, the side slope is m = 1, and it is applied to design the energy dissipation construction for the Na San irrigation system (Son La).

CONCLUSIONS

1. General conclusions of the thesis

+ Overview of research results of the hydraulic jump in the prismatic channels (rectangular and trapezoidal channel), using to a basis for analyzing and establishing relationships between the geometrical features of the jump.

+ Establishing the new critical depth equation (3.8).

+ Solving the Navier-Stokes equation, equation (3.36) has been determined for the calculation of the sequent depth (y_r) and the ratio of the momentum coefficient (k) of the jump.

+ Experimental research for the jump in an the horizontal trapezoid channel with a side slope m = 1, has determined the theoretical equation for the sequent depth (3.36) and the k = 0.92. Table 3.9 and the Figure 3.5 have been built to determine the sequent depth ratio of the jump in the range Fr_{D1} = 4.0 ÷ 9.0 and M_1 = 0 ÷ 1.

+ Establishing a semi-empirical equation (3.50) on determining the roller length of the jump in the trapezoidal channel (flat bottom and the side slope m = 1).

+ The study has additionally developed 02 new empirical equations for calculating the sequent depth (3.39) and the roller length (3.53) of the jump, combined with theoretical and semi-empirical equations to have a basis for selecting design values.

+ For the first time, applying the "Random Forest" model to predict the geometric features of the jump in the trapezoidal channel, the analytical results are equivalent to other research methods.

+ Developing a process to apply the proposed equations to determine the geometrical features of the jump in the trapezoidal channel and calculating an energy dissipator construction in practice.

2. Shortcomings and further researches

+ The study is currently conducted on a type of physical model, so the research results still have certain limitations.

+ The next research will enhance more for physical models and determine the k suitable for all types of the trapezoidal channels.

+ Study the relationship between the roller length (L_r) and the jump length (L_J).

+ Application to expand Machine Learning models in the study of the geometric characteristics of the hydraulic jump.

LIST OF AUTHOR'S PUBLISHED ARTICLES

1. Le Van Nghi, **Nguyen Minh Ngoc** (2019). Study to determine the length of the hydraulic jump in the trapezoidal channel. *The 22nd National Conference on Hydraulic Mechanics. Thanh Nien Publishing House . ISBN 978-604-979-703-3, pp 606-617.*

2. **Nguyen Minh Ngoc**, Le Van Nghi, Pham Hong Cuong (2020). Application correlation algorithm to create a new critical depth equation for gradually varied flow in trapezoidal channel using teaching–learning and studying. *Journal of Hydraulic Structures (JHS), 2020; 6(2):80-94. DOI: 10.22055/jhs.2020.34372.1141 (in Publons system of WOS).*

3. Pham Hong Cuong, **Nguyen Minh Ngoc**, Le Quang Hung (2020). Hydraulic jump and factors affecting the geometrical characteristics of the jump in a prismatic channel. *Journal of Water Resources Science and Technology . No 62, pp 70 – 76.*

4. **Nguyen Minh Ngoc** (2021). Creating new critical depth formula of trapezoidal channe*l*. *Science Journal of Architecture & Construction. No 42 – 08.2021, pp 45 – 51.*

5. **Ngoc, N. M.,** Cuong, P. H., Son, T. T, Nam, N. V., Phong, N. T. (2022). Experimental study of the hydraulic jump length in a smooth trapezoidal channel. *Scientific Review Engineering and Environmental Sciences. Vol 31 (1), p.63–76; DOI:10.22630/srees.2334 (Scopus, Q4).*

6. **Nguyen Minh Ngoc,** Pham Hong Cuong, Bui Hai Phong (2022). Prediction of the conjugate depth of the hydraulic jump in the trapezoidal channel using Random Forest regression. *Journal of Military Science and Technology (JMST) - Accepted*